

Brief communication

Interfacial friction correlations for the two-phase flow across tube bundle

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Received 21 February 2006; received in revised form 9 August 2006

Keywords: Two-phase flow; Tube bundle; Interfacial friction

1. Introduction

The two-fluid model is an advanced predictive tool for liquid and gas two-phase flows in engineering applications. It is based on the mass, momentum and energy balance equations for each phase (Ishii, 1987). This approach enables the prediction of important non-equilibrium phenomena of two-phase flow, such as differences in liquid and gas phase velocities and temperatures. The prediction of liquid and vapour phase velocity difference is important for two-phase flows in large shell sides of steam generators and kettle reboilers, where even different gas and liquid velocity directions exist. The liquid phase circulates in the upward or lateral direction across the tube bundles and in the downward direction in the free passages between the tube bundles and the vessel walls, while the vapour flows mainly in the upward direction. The liquid and vapour separation occurs at the two-phase mixture swell level (at the swell level vapour flows to the steam dome, while liquid recirculates). The vapour void fraction depends both on the boiling intensity and liquid and vapour velocity difference. The vapour and liquid velocity fields and the related void fractions are important for the operational characteristics of steam generators and kettle reboilers: the void fraction determines the swell level position, both void fraction and phases' velocity fields determine the two-phase flow structure and heat transfer from the tubes to the boiling mixture, and a possible occurrence of the critical heat transfer due to tube dry-out. These phenomena and their effects were experimentally and numerically investigated for instance by Ageev et al. (1987), Edwards and Jensen (1991), Groburov and Zorin (1994), King and Jensen (1995), Urban et al. (2002), and Gebbie and Jensen (1997). The accuracy of the two-fluid model prediction of velocity and void fraction strongly depends on the reliability of the applied model for the momentum interfacial transfer due to liquid–vapour friction. The momentum transfer due to interfacial friction depends on the topology

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of the interface and on the interfacial shear stress. Therefore, the interfacial friction force per unit volume in the two-fluid model is expressed as

$$\vec{F}_{21} = a_{21} \vec{\tau}_{21}, \quad (1)$$

where a_{21} is the interfacial area concentration in (m^2/m^3) and $\vec{\tau}_{21}$ is the interfacial shear stress. For dispersed vapour in liquid flow the above relation is expressed in the form

$$\vec{F}_{21} = \frac{3}{4} \alpha_2 \rho_1 \frac{C_D}{D_p} |\vec{u}_2 - \vec{u}_1| (\vec{u}_2 - \vec{u}_1), \quad (2)$$

where α_2 is the gas-phase void fraction within the control volume, ρ_1 is the liquid density, \vec{u}_k stand for velocity vector ($k = 1$ represents liquid and $k = 2$ gas phase), C_D is the interfacial drag coefficient and D_p is the diameter of the dispersed bubble.

A number of interfacial drag correlations have been developed for two-phase flows in pipes, but only [Rahman et al. \(1996\)](#) proposed one for two-phase flow across tube bundles. This correlation has the form

$$C_D = (C_{D,u}^{-4} + C_{D,l}^{-4})^{-0.25}, \quad (3)$$

where the indices u and l denote respectively correlations for “upper” and “lower” regions in a plot of the interfacial drag coefficient versus the Reynolds number. Two regions are arbitrarily separated by [Rahman et al.](#) at $C_D \approx 4$. As shown by (3), the correlations for the upper and lower region are combined using the [Churchill and Usagi \(1972\)](#) expression. The interfacial drag coefficient for both regions is defined with the power law function of the tube bundle porosity ψ and the two-phase mixture Reynolds number

$$C_{D,u \text{ or } l} = e^E \psi^\beta Re^n. \quad (4)$$

In (4) e is the base of natural logarithm, and the Reynolds number is

$$Re = \frac{\rho_m v_r \delta}{\mu_1}, \quad (5)$$

where v_r is the difference between gas and liquid phase velocity, δ is the product of the tubes’ transverse pitch and the tubes bundle porosity, and ρ_m and μ_1 are respectively the two-phase mixture density and the liquid dynamic viscosity. Different values of constants E , β and n are determined for the lower ($C_{D,l} < 4$) and the upper region ($C_{D,u} > 4$), as well as for in-line and staggered tube arrangements in the bundle. The correlation expressed by Eqs. (3) and (4) is derived on the basis of experimental data of air-water upward flows across horizontal tube bundles obtained by [Dowlati et al. \(1990, 1992a\)](#). These measurements were performed for both in-line and staggered tubes arrangements in the bundle, which consisted of 20 rows of tubes. A single-beam gamma densitometer was used to measure the void fraction at various elevations in the bundle. The error of the void fraction measurements was ± 0.05 ([Dowlati et al., 1992a](#)). One measurement gives a mean value of void fraction across a row at certain elevation. A nearly constant value of the void fraction was measured along the bundle height for each experimental run ([Dowlati et al., 1990](#)). Such fully developed flow conditions are a result of the adiabatic conditions of air and cold-water flow without a phase change. Also, a high resistance of the bundle to the two-phase mixture flow leads to the establishment of a fully developed flow nearly after the first row of tubes. [Rahman et al. \(1996\)](#) introduced this condition in their hydraulic model. They wrote the momentum balance equation for fully developed two-phase flow conditions and calculated explicitly values of the interfacial drag coefficient C_D by introducing the measured data from many experimental runs. Also, they argued that the contact between the air and tubes is negligible, and they neglected the momentum loss due to tubes resistance to gas flow in the momentum equation.

A set of two interfacial friction correlations for two-phase flow across tube bundles was proposed and applied in multidimensional Computational Fluid Dynamics simulations of horizontal steam generators by [Stevanovic et al. \(2002a,b\)](#), [Stosic and Stevanovic \(2002\)](#) and kettle-reboilers by [Pezo et al. \(2006\)](#). In this approach two patterns of two-phase flow across the tube bundle are observed: bubbly flow for void fractions lower than 0.3, and churn turbulent and annular flow for void fractions higher than 0.3. Since there are no experimental data on interfacial area concentration in two-phase flow across tube bundles, and it cannot be practically estimated, the ratio of the interfacial drag coefficient and the dispersed bubble diameter

C_D/D_p in Eq. (2) is correlated. For bubbly flow, the modified form of the Ishii and Zuber (1979) correlation, developed for two-phase pipe flow is applied. For churn-turbulent flow, the new correlation is developed with functional dependence on the void fraction in a similar form as it was applied in the CATHARE code by Rousseau and Houdayer (1983).

In the works of Stevanovic et al. (2002), Stosic and Stevanovic (2002) and Pezo et al. (2006) good predictions of spatial void fraction distributions were obtained for two-phase flow conditions in complex geometries of large-scale vessels with internal obstacles. But, the complexity of simulated conditions and the applied modelling approach and numerical solution procedure impose many influences on the void fraction prediction. Therefore, the influence of the interfacial friction correlation within the complex model can be superimposed with other effects that may (or may not) lead to correct final result in the void fraction prediction.

In the present work, the proposed set of two interfacial friction correlations is validated for the partial effect tests of one-dimensional adiabatic two-phase flow across tube bundles performed by Dowlati et al. (1990, 1992a,b). In these experiments the influence of other complex conditions that can be encountered in engineering practice, such as boiling, presence of obstacles, recirculation effects, is excluded. Gas and liquid momentum balance equations are written for fully developed steady-state flow, since they are appropriate for the Dowlati et al. experimental runs. The sum of the gas and liquid momentum equations results in a non-linear algebraic equation with the two-phase void fraction as the only unknown variable, which is calculated with the bisection method for finding the real zeros of a continuous function (Hamming, 1986). Also, modelling predictions of the drift velocity are compared with values derived from the experimental data, with satisfactory agreements. It should be emphasised that the proposed correlations are validated by solving both liquid and gas phase momentum equations and by taking into account the tube bundle resistance to the liquid and gas flows, while the Rahman et al. (1996) correlation (Eqs. (3) and (4)) was developed by calculating the interfacial friction only from the gas phase momentum balance without the inclusion of the tubes resistance to the gas flow, an effect that is not negligible in case of high void two-phase flows. Also, calculated drift velocities show a dependence on the two-phase mass flux and void fraction. Hence, the drift velocity in two-phase flows across tube bundles cannot be assumed constant, as suggested by Dowlati et al. (1992b).

2. Problem statement

The adiabatic air–water upward flow across horizontal tubes with in-line or staggered arrangements in the bundle is shown in Fig. 1. The transverse tube pitch is P_t and the longitudinal P_l . The tubes volume fraction within the control volume depicted in Fig. 1 is α_3 , while the two-phase mixture volume fraction is denoted as porosity ψ . The flow conditions simulated in this study, Table 1, are the same as in the experimental runs of

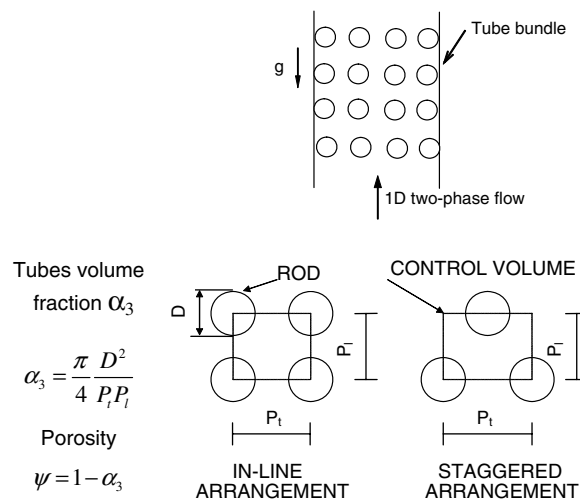


Fig. 1. Adiabatic air–water two-phase flow across horizontal in-line and staggered tube bundles.

Table 1
Experimental conditions of Dowlati et al. (1990, 1992a,b)

Tubes arrangement in the bundle	In-line		Staggered	
Pitch-to-diameter ratio, P/D	1.3	1.75	1.3	1.75
Tube diameter, D (mm)	19.05	12.7	19.05	12.7
Mass flux in the clearance between tubes, G_{\max} ($\text{kg m}^{-2} \text{s}^{-1}$)	27–818	90–542	92–795	56–538
Mixture quality, x	0–0.33	0–0.08	0–0.15	0–0.13

Dowlati et al. (1990, 1992a,b). Because the pressure drop was moderate and the experiments were conducted adiabatically, the mean flow was not accelerating and, hence, a constant void fraction profile is observed after the first row of tubes. The local flow around tubes in the bundle is at least two-dimensional, but the dominant flow direction within the whole volume of the bundle is upward; hence, with the aim at predicting the mean two-phase flow parameters within the bundle, a one-dimensional flow is presently assumed.

3. Modelling approach

The two-phase flow across the tube bundles is assumed as fully developed, one-dimensional, steady state, adiabatic, and upwardly directed. It is modelled with the momentum balance equations for the liquid phase

$$\alpha_1 \frac{\partial p}{\partial z} = -\alpha_1 \rho_1 g + F_{21} - F_{31} \quad (6)$$

and the gas phase

$$\alpha_2 \frac{\partial p}{\partial z} = -\alpha_2 \rho_2 g - F_{21} - F_{32}, \quad (7)$$

where the left hand side terms represent the pressure force acting on the liquid (Eq. (6)) and gas phase (Eq. (7)), the first terms on the right hand side represent gravity forces, F_{21} is the gas–liquid interfacial friction force and F_{31} and F_{32} are liquid–tubes and gas–tubes flow resistance, respectively, all written per unit volume. The calculation of F_{31} and F_{32} terms in (6) and (7) is presented in Appendix A. The pressure losses due to the tube skin friction and form drag in the bundle are calculated for each phase by applying correlations developed for one-phase flow and by multiplying them with the phase volume fraction. The sum of liquid and gas phase pressure drops results in the total frictional pressure drop calculated with the homogeneous two-phase model (Eq. (A.11)).

The tube bundle is assumed as porous media and the following volume fraction balance is written for the liquid (α_1), gas (α_2) and tubes (α_3) for each control volume

$$\alpha_1 + \alpha_2 + \alpha_3 = 1. \quad (8)$$

The gas-phase void fraction in the two-phase mixture is calculated as

$$\varphi = \frac{\alpha_2}{\alpha_1 + \alpha_2} \quad (9)$$

and the volume fractions of the liquid and gas phase within the porous media are derived from (8) and (9) as

$$\alpha_2 = \varphi(1 - \alpha_3), \quad (10)$$

$$\alpha_1 = (1 - \varphi)(1 - \alpha_3). \quad (11)$$

By substituting the pressure change along the bundle height $\partial p/\partial z$ from (6) into (7) and introducing relations (10) and (11) the following balance of volume forces is obtained:

$$\varphi(1 - \alpha_3)(\rho_1 - \rho_2)g - \left(1 + \frac{\varphi}{1 - \varphi}\right)F_{21} + \frac{\varphi}{1 - \varphi}F_{31} - F_{32} = 0. \quad (12)$$

The interfacial friction force F_{21} is calculated with Eq. (2), where the gas and liquid phase velocity vectors have only vertical components u_2 and u_1 . The gas and liquid phase velocities are calculated from the mass balance equations for the two-phase flow through the porous media and by applying experimentally known or assumed values of the two-phase flow mass flux G_{\max} and the mixture quality x . The liquid and gas phase velocities are calculated from the following expressions:

$$u_1 = \frac{(1-x)G}{\rho_1(1-\varphi)(1-\alpha_3)}, \quad (13)$$

$$u_2 = \frac{xG}{\rho_2\varphi(1-\alpha_3)}. \quad (14)$$

The total two-phase mass flux in the porous media is calculated from the experimental value of the maximum two-phase flow mass flux in the clearance between adjacent tubes, hence

$$G = G_{\max} \left(1 - \frac{1}{P_i/D} \right). \quad (15)$$

The following correlations are proposed for the calculation of the ratio of the interfacial drag coefficient C_D and the dispersed bubble diameter D_p . For bubbly flow ($\varphi \leq 0.3$) the Ishii and Zuber (1979) correlation, developed for the distorted bubble two-phase flow inside tubes, is adopted for the calculation of C_D/D_p . In order to obtain the best agreement of the calculated void fraction results with the experimental data of Dowlati et al. (1990, 1992a,b) the original correlation is multiplied with 0.4. This reduction could be attributed to the fact that due to the presence of tubes in the bundle, bubbles deviate more from the spherical shape and they coalesce more, leading to lower value of the interfacial drag coefficient

$$\frac{C_D}{D_p} = 0.267 \left(\frac{g\Delta\rho}{\sigma} \right)^{1/2} \left\{ \frac{1 + 17.67f(\varphi)^{6/7}}{18.67f(\varphi)} \right\}^2 \quad (16)$$

where $\Delta\rho$ is the difference between liquid and gas phase density, g is the gravity, σ is the surface tension coefficient and

$$f(\varphi) = (1 - \varphi)^{1.5}. \quad (17)$$

For churn-turbulent flows ($\varphi > 0.3$), a new correlation is proposed

$$\frac{C_D}{D_p} = 1.487 \left(\frac{g\Delta\rho}{\sigma} \right)^{1/2} (1 - \varphi)^3 (1 - 0.75\varphi)^2, \quad (18)$$

where the dependence on the mixture void fraction φ has the same functional form as the CATHARE code correlation presented by Rousseau and Houdayer (1983) for the interfacial friction in the transitional churn turbulent, as well as in the separated annular two-phase flow pattern. According to Eq. (18) an increase of the void fraction leads to a sharp decrease of C_D/D_p , as it is shown in Fig. 2 for air–water mixture at atmospheric pressure. This characteristic dependence is attributed to the decrease of the gas–liquid interfacial area concentration, i.e. to the increase of D_p with void fraction increase. This conclusion is in accordance with the fact that the interfacial area decreases as void fraction increases in a transition from bubbly to churn and annular flow patterns. The term $(g\Delta\rho/\sigma)^{1/2}$ in Eqs. (16) and (18) is proportional to the reciprocal of the maximum stable rising bubble diameter (Clift et al., 1978). The correlations (16) and (18) depend on the liquid and gas phase thermo-physical properties; hence, the curve of the same shape as in Fig. 2, but shifted along the vertical axis is presented by Pezo et al. (2006) for the refrigerant R113 fluid.

The present model is accompanied by the calculation of the drift flux velocity

$$u_{dj} = u_2 - j, \quad (19)$$

where j is the two-phase superficial velocity defined as

$$j = (1 - \varphi)u_1 - \varphi u_2. \quad (20)$$

Substituting Eqs. (13) and (14) into Eq. (19) it is obtained

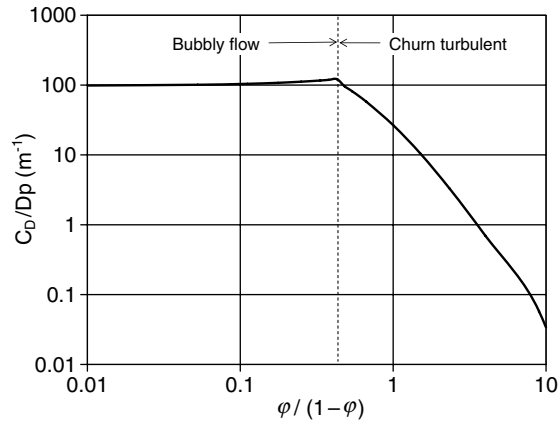


Fig. 2. Ratio of interfacial friction coefficient C_D and dispersed bubble diameter D_p versus void fraction for two-phase flow across a tube bundle calculated with (16) for bubbly and (18) for churn turbulent flow.

$$u_{dj} = \frac{G}{(1 - \alpha_3)} \left[\frac{x}{\rho_2} \left(\frac{1}{\phi} - 1 \right) - \frac{1 - x}{\rho_1} \right]. \quad (21)$$

Results of the drift flux velocity calculated with Eq. (21) are presented and compared with results of previous models in the next section.

The steady state and fully developed two-phase flow across a tube bundle is determined by the two-phase flow mass flux (usually the maximum value G_{\max} is used), the mixture quality x , and the geometry of the tube bundle. The void fraction ϕ is calculated as the zero of function (12) with the bisection method (Hamming, 1986). It should be mentioned that all volume forces F_{21} , F_{31} , and F_{32} and phase velocities (defined with Eqs. (13) and (14)) are functions of void. Further details of the model and the calculation procedure are presented in Simovic (2001) and Ocokoljic (2003).

4. Results and discussions

Calculated values of void fraction are compared with 180 measured data in Fig. 3. The achieved agreement is within the void fraction deviation of ± 0.08 , a range which is of the same order as the error of ± 0.05 of void fraction measurements by Dowlati et al. (1992a). The results in Fig. 3 show the same accuracy in predicting void fraction for both in-line and staggered tubes. At the same time, none of the parameters in the correlations

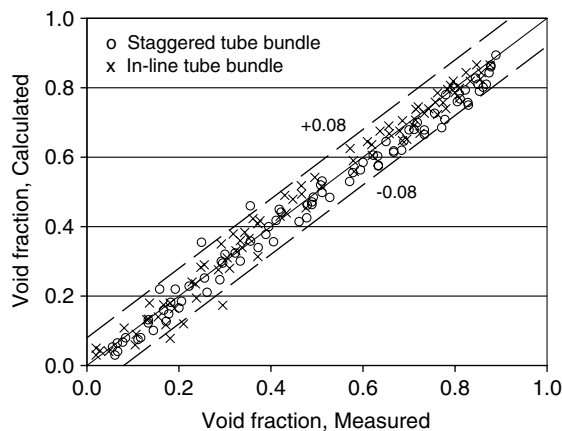


Fig. 3. Calculated versus measured void fractions in air–water flow across horizontal bundles with in-line and staggered tubes arrangements.

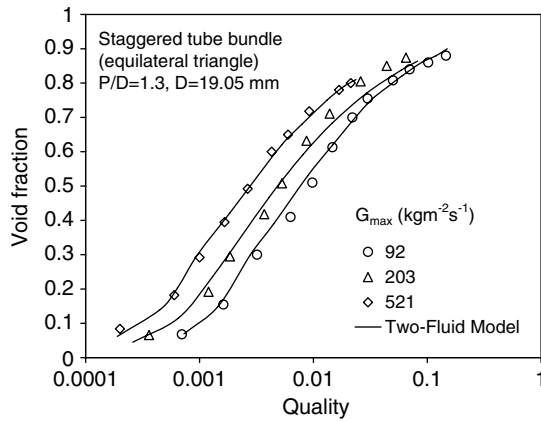


Fig. 4. Comparison of measured void fraction and numerical simulation results for air–water flow across horizontal tube bundle.

(16) and (18) depends on the tube bundle arrangement, while in the correlation (4) of Rahman et al. (1996) there is a need to prescribe some experimental parameters for in-line and staggered tubes arrangements in the bundle. This indicates the more general nature of the correlations proposed in this study. Fig. 4 shows the dependence of the void fraction on the mixture quality. The results for the staggered tube bundle are presented, but this dependence is typical for both in-line and staggered tube arrangements and different pitch-to-diameter ratios. Both experimental and modelling results show a void fraction dependence on the two-phase flow mass flux G , as well as “S” type dependence of the void fraction on the mixture quality (the void fraction change is slowed in the range of high and low mixture qualities). It should be noted that the same results as presented in Fig. 4 are obtained by Pezo et al. (2006) by applying the same correlations for the interfacial drag coefficient, but a much more complex numerical model based on the two-dimensional differential governing equations of the two-phase two-fluid model and the IPSA solving algorithm (Spalding, 1983).

In the two-fluid modelling approach, the closure law for the momentum transfer due to interfacial friction determines the gas and liquid phase velocity difference. The level of non-equilibrium between the gas and liquid phase velocity is usually expressed through the drift velocity. Therefore, the proper prediction of the drift velocity is also a good verification for the reliability of the applied closure law for the gas and liquid phase interfacial momentum transfer. Having this in mind, the experimental data of Dowlati et al. (1990, 1992a,b) are analyzed and drift velocities for these experiments are calculated from Eq. (20). Also, drift velocities are calculated from the present void data from the two-fluid model predictions. These drift velocities calculated from the experimentally measured data and from the two-fluid model results are presented and compared in Fig. 5. It is shown that the drift velocity depends on the void fraction and on the two-phase flow mass flux,

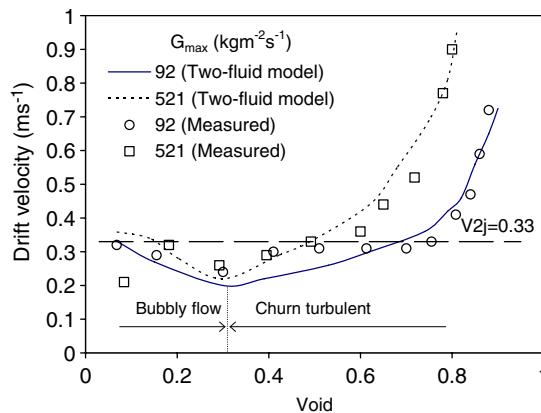


Fig. 5. The drift velocity dependence on the void fraction (the constant value 0.33 from Dowlati et al., 1992b).

and its value could not be assumed as being constant. The constant value of 0.33 ms^{-1} is shown in Fig. 5 as a result of Dowlati et al. (1992b). They obtained this result by correlating their data with the experimental relation

$$u_2 = C_0 j + u_{d2}, \quad (22)$$

where C_0 is the distribution parameter predicted as 1.1035 and u_{d2} is the drift velocity predicted as 0.33 ms^{-1} . Obviously, their averaging procedure could not give detailed insight into drift velocity dependence on void. Some other investigators also adopted constant drift–flux velocity in their calculations, such as Groburov and Zorin (1994) in the analyses of a three-dimensional shell side two-phase flow in a horizontal steam generator of a nuclear power plant. Also, the shapes of the presented drift velocity curves in Fig. 5 indicate that the drift velocity depends on the two-phase flow pattern. There is a minimum of the drift velocity for the void fraction value of approximately 0.3, which indicates the transition from the bubbly to churn-turbulent flow pattern. The assumption of these two flow patterns is also supported by the experimental evidence of Dowlati et al. (1992b) that nearly all test conditions resulted in dispersed bubbly or churn-turbulent bubbly flow.

5. Conclusions

A set of two gas–liquid interfacial friction correlations is proposed for application within the porous media based two-fluid models of two-phase flows across tube bundles. Both correlations predict the ratio of drag coefficient to the distorted bubble diameter, i.e. C_D/D_p . For void fraction values lower or equal to 0.3, which correspond to bubbly flow, the proposed correlation is based on the modified Ishii and Zuber (1979) correlation. For void fraction values higher than 0.3, which correspond to churn or annular flow patterns, the proposed correlation predicts a sharp decrease of C_D/D_p with increasing void fraction, as it was applied in the CATHARE code correlation (Rousseau and Houdayer, 1983). The proposed correlations are validated for the 180 experimental runs of upward air–water flows across in-line and staggered tube bundles with different pitch-to-diameter ratios performed by Dowlati et al. (1990, 1992a,b). Good agreement is obtained between the calculated and measured void fractions, as well as the drift velocities based on the calculated and measured void fractions. The accuracy of the void fraction predictions is in the same range as the void fraction measurements. The analysis shows that the drift velocity depends on the void fraction and the two-phase flow mass flux, and its value cannot be assumed constant, as it was reported in previous studies. The geometry and conditions of the simulated and analysed experimental runs provide steady-state, fully developed 1-D flow that is dominated by interfacial friction. Also, the developed model is suited to these simplified experimental conditions. It is expressed in the form of non-linear algebraic equation with the void fraction as the unknown parameter, which is easily calculated with the bisection method. This approach minimizes the modelling and calculation errors, and eliminates the diffusion errors that are encountered in the solution of the governing differential equations. Hence, although good predictions of void fraction were obtained with the proposed interfacial friction correlations in previous multidimensional simulations of steam generators and kettle reboiler shell side thermal-hydraulics (Stevanovic et al., 2002a,b; Stosic and Stevanovic, 2002 and Pezo et al., 2006), there was a need for the analyses presented in this paper in order to exclude the ambiguity that could be introduced by multiple effects of the real conditions in steam generating equipment.

Acknowledgment

The research described in this paper was partially funded by the Ministry of Science and Environmental Protection of the Republic of Serbia (project 144022).

Appendix A. Frictional pressure drop

The frictional pressure drop of the liquid phase flow is correlated with the Euler number Eu , taking into account the volume occupied by the liquid ($1 - \varphi$)

$$\Delta p_1 = Eu_1 \rho_1 u_1^2 (1 - \varphi). \quad (\text{A.1})$$

For the gas phase a similar equation is written

$$\Delta p_2 = Eu_2 \rho_2 u_2^2 \varphi. \quad (\text{A.2})$$

The Euler numbers Eu_i , $i = 1, 2$, are calculated with the experimental correlations proposed by Isachenko et al. (1980) for one-phase flows across tube bundles. They are written here for completeness of the presented model in the following reduced form applicable to equilateral staggered and in-line tubes arrangements ($P = P_t = P_l$):

Staggered tube bundle

$$Eu_k = 1.4(z + 1)Re_k^{-0.25} \quad \text{for} \quad \frac{1 - \frac{D}{P}}{\frac{D}{P} - 1} \leq 0.53, \quad (\text{A.3})$$

$$Eu_k = 1.93(z + 1) \sqrt{\frac{1 - \frac{D}{P}}{\frac{D}{P} - 1}} Re_k^{-0.25} \quad \text{for} \quad \frac{1 - \frac{D}{P}}{\frac{D}{P} - 1} > 0.53. \quad (\text{A.4})$$

In-line tube bundle

$$Eu_k = 0.265 \left(\frac{\frac{P}{D} - 0.8}{\frac{D}{P} - 1} \right)^{2.5} z Re_k^m \quad \text{for} \quad \frac{\frac{P}{D} - 0.8}{\frac{D}{P} - 1} \leq 1, \quad (\text{A.5})$$

$$Eu_k = 0.265 \left(\frac{\frac{P}{D} - 0.8}{\frac{D}{P} - 1} \right)^2 z Re_k^m \quad \text{for} \quad \frac{\frac{P}{D} - 0.8}{\frac{D}{P} - 1} > 1, \quad (\text{A.6})$$

where z represents the number of tube rows in the direction of flow, and k denotes liquid ($k = 1$) or gas ($k = 2$) phase. The exponent m in (A.5) and (A.6) is calculated as follows:

$$m = -0.133 \quad \text{for} \quad \frac{P}{D} \geq 1.24, \quad (\text{A.7})$$

$$m = 0.867 \left(\frac{P}{1.24D} \right)^{0.7} - 1 \quad \text{for} \quad \frac{P}{D} < 1.24, \quad (\text{A.8})$$

$$\text{the relation between } Eu \text{ and the frictional loss coefficient } f \text{ is : } f = 2Eu. \quad (\text{A.9})$$

The momentum loss due to tube bundle resistance to two-phase flow is calculated as

$$F_{3k} = (1 - \alpha_3) \Delta p_k / \Delta z, \quad (\text{A.10})$$

where Δz is the height of the computational cell. It can be noted that the sum of (A.1) and (A.2), with the assumption that $f_1 \cong f_2 \cong f$, is the well known expression for the frictional pressure drop in the homogeneous two-phase flow

$$\Delta p = f \frac{G_{\max}^2}{2\rho_1} \left[1 + x \left(\frac{\rho_1}{\rho_2} - 1 \right) \right], \quad (\text{A.11})$$

$$\text{where the mass flux } G_{\max} \text{ is : } G_{\max} = (1 - \varphi) \rho_1 u_{1,\max} + \varphi \rho_2 u_{2,\max}. \quad (\text{A.12})$$

The mass flux G_{\max} and the Reynolds number Re_k are calculated with the maximum phase velocities, $\dot{u}_{k,\max}$, which are defined for the minimum flow area in the clearance between two adjacent tubes. A characteristic dimension for the Re_k calculation is the tube's outer diameter.

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